

# Drag and Diffusion coefficients in extreme scenarios of temperature and chemical potential

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## Abstract

A comparative study of high and zero temperature plasma for the case of damping rate, drag and diffusion coefficients have been presented. In each of these quantities, it is revealed how the magnetic interaction dominates over the electric one at zero temperature unlike what happens at high temperature.

## 1 Introduction

The study of nuclear matter under extreme conditions has been an active field of research for the past few decades. In particular the creation of quark-gluon plasma at high temperature has drawn special interest both in the theoretical and experimental fronts in view of different experimental programs like RHIC, LHC etc. The other domain of QCD phase diagram where chemical potential is considered to be much higher than the temperature is less explored. This is the region of interest for the upcoming experiments on compressed baryonic matter to be performed at FAIR/GSI. The high density QCD or QED plasma has also its relevance to astrophysics. In particular it is known that the ultradegenerate plasma might exist in the core of the neutron stars.

In plasma, particles propagation get modified through the interaction with the surrounding medium (quasiparticles). These quasiparticles are the relevant degrees of freedom in terms of which the dynamics of the plasma has to be understood. In this work we focus on three quantities *viz* the damping rate, drag and momentum diffusion coefficients of the plasma. The latter two are intimately connected with the former as exposed in the current work. The study of quasiparticle damping rate ( $\gamma$ ) as well as the transport coefficients like drag and diffusion coefficients ( $\eta, \mathcal{B}$ ) in case of high temperature plasma have been studied for last decades [1, 2, 3], whereas computation of these coefficients in case of degenerate plasma is relatively unexplored field of research. Calculation of all these quantities, as is known, suffer from infrared divergences for bare Coulomb or magnetic interaction. How these divergences are handled in the finite temperature by using the techniques of hard-thermal loop (HTL) [4] have been well studied. We also encounter similar divergences at zero temperature where the hard-dense loop (HDL) [5] corrected propagator for the intermediate

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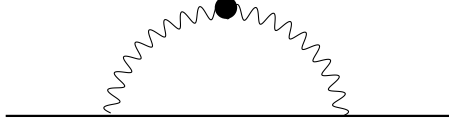


Figure 1: Fermion self-energy with resummed photon propagator.

boson is used to remove such divergences. Here, we discuss the departure of behavior of high temperature plasma from the zero temperature one. To reveal these differences further and understand the dynamics evolved in each cases we first focus on  $\gamma$  subsequently we discuss  $\eta$  and  $\mathcal{B}$ .

## 2 Quasiparticle damping rate in hot and dense plasma

The quasiparticle damping rate in the field theoretical language can be written in the terms of self-energy. Assuming the energy of the quasiparticle to be hard  $E \sim \mu$ ,

$$\gamma \equiv -\frac{1}{4p} \text{tr} [\not{p} \text{Im} \Sigma(p_0 + i\epsilon, \mathbf{p})] |_{p_0=p}, \quad (1)$$

$\Sigma$  is the Fermion self-energy,

$$\Sigma(P) = e^2 T \sum_s \int \frac{d^3 q}{(2\pi)^3} \gamma_\mu S_0(P-Q) \gamma_\nu \Delta_{\mu\nu}(Q). \quad (2)$$

$S_0(P-Q)$  and  $\Delta_{\mu\nu}(Q)$  are the free fermion propagator and dressed photon propagator respectively. Detail structures of these propagators can be found in [6]. Explicit evaluation of Eq.(1) gives,

$$\begin{aligned} \gamma &= \frac{\pi e^2}{E} \int \frac{d^3 q}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{dk_0}{2\pi} \rho_f(k_0) \int_{-\infty}^{\infty} \frac{dq_0}{2\pi} \\ &\times (1 + n(q_0) - \bar{n}(k_0)) \delta(E - k_0 - q_0) \\ &\times [p_0 k_0 + \mathbf{p} \cdot \mathbf{k} + m^2] \rho_l(q_0, q) \\ &+ 2[p_0 k_0 - (\mathbf{p} \cdot \hat{\mathbf{q}})(\mathbf{k} \cdot \hat{\mathbf{q}}) - m^2] \rho_t(q_0, q), \end{aligned} \quad (3)$$

$\rho_{l,t}$  are the imaginary part of the photon propagator. The calculation of the quasiparticle damping rate is plagued with infrared divergences in case of soft photon momentum exchange. To remove the divergences one can in principle use the Braaten and Yuan's prescription (BY) [7]. In this prescription an intermediate momentum scale  $q^*$  is introduced, so that for the

exchanged momentum region  $q > q^*$  one can use the bare propagator and in the other domain where  $q < q^*$  HTL/HDL resummed propagator has to be incorporated. In case of high temperature relativistic plasma  $\gamma$  has been studied earlier in [1]. The phase space factor in the high temperature limit can be written as  $(1 + n(q_0) - \bar{n}(k_0)) \sim T/q_0$ , with this and HTL propagator the quasiparticle damping rate remains divergent in the transverse sector giving [1],

$$\gamma_t = \frac{g^2 T}{2\pi^2} \int_0^{q^*} \frac{dq}{q} \tan^{-1} \left( \frac{3\pi\omega_p^2}{4q^2} \right), \quad (4)$$

while for the longitudinal case we have [1],

$$\gamma_l = g^4 T^3 / m_D^2. \quad (5)$$

In the zero temperature limit the phase space factor takes the following form  $1 + n(q_0) = \theta(q_0)$  and  $\bar{n}(k_0) = \theta(\mu - E - q_0)$  due to Pauli blocking. In this case ( $T = 0, \mu \neq 0$ ) one can eventually show that with the help of the Pauli blocking and HDL propagator finite  $\gamma$  can be obtained [8, 9],

$$\begin{aligned} \gamma_l &= \frac{e^2 \nu^2 m_D}{64}, \\ \gamma_t &= \frac{e^2 \nu m_D}{24\pi}. \end{aligned} \quad (6)$$

Here,  $|E - \mu|/m_D = \nu \ll 1$ , which means the scattering of quasiparticles take place very close to the Fermi surface and the results are finite.

### 3 Quasiparticle drag and diffusion coefficients in hot and dense plasma

The quasiparticle drag coefficient is related to the energy loss of the particle,

$$\eta = -\frac{1}{E} \frac{dE}{dx} \quad (7)$$

where,  $-dE/dx$  is the particle energy loss which can be obtained by weighting the interaction rate with the energy transfer per scattering. Hence, one can obtain the expression for the drag coefficient as follows,

$$-\frac{dE}{dx} = \int d\Gamma \omega, \quad (8)$$

$\Gamma$  is the interaction rate. In case of high temperature plasma with bare propagator in  $2 \rightarrow 2$  scattering process  $\eta$  shows logarithmic divergence  $\frac{e^4 T^2}{24\pi E} \int \frac{dq}{q}$  whereas in case of ultradegenerate plasma the divergence is even more worse  $\frac{e^2 (E-\mu)^3 m_D^2}{16\pi E} \int \frac{dq}{q^4}$ . In both the cases BY prescription can be used to show that finite results can be obtained [6]. Even the algebraic divergence of ultradegenerate plasma can be removed with the help of the

HDL propagator. We eventually have to evaluate the following expression in two extreme cases  $T \neq 0, \mu = 0$  and  $T = 0, \mu \neq 0$ ,

$$\begin{aligned} \left(-\frac{dE}{dx}\right) &\simeq \frac{e^2 m_D^2}{4\pi} \int dq_0 dq (1 + n(q_0) - \bar{n}(E - q_0 - \mu)) \\ &\times \left\{ \frac{q_0^2}{\left\{2 \left[q^2 + m_D^2 Q_l\left(\frac{q_0}{q}\right)\right]^2 + \frac{m_D^4 \pi^2 q_0^2}{2q^2}\right\}} \right. \\ &\left. + \frac{q_0^2}{\left[2q^2 + m_D^2 Q_t\left(\frac{q_0}{q}\right)\right]^2 + \frac{m_D^4 \pi^2 q_0^2}{4q^2}} \right\}, \end{aligned} \quad (9)$$

where,

$$Q_l(x) = 1 - \frac{x}{2} \ln \frac{1+x}{1-x}, \quad Q_t(x) = -Q_l(x) + \frac{1}{1-x^2}. \quad (10)$$

The denominator of the Eq.(9) can be expanded in powers of  $x$  and for the longitudinal exchange the denominator can be replaced by  $(q^2 + m_D^2)^2$  and for the transverse interaction  $q^4 + (\pi^2 m_D^4 x^2)/4$ ,

In the high temperature plasma the phase space factor can be approximated as  $T/q_0$  as mentioned earlier, with this we obtain the result for  $\eta$  [3],

$$\eta \simeq \frac{e^4 T^2}{36\pi E} \left( -\frac{1}{2} + \frac{3}{2} \log \left| \frac{q_{\max}}{m_D} \right| + \frac{1}{2} \log \left| \frac{2}{\sqrt{\pi}} \right| \right), \quad (11)$$

where,  $q_{\max}$  can be set by the kinematics of the problem. In the limit where  $T = 0$  and  $\mu \neq 0$  the drag coefficient turns out to be [6],

$$\eta \simeq \frac{e^2 m_D^2 \nu^2}{48\pi E} + \frac{e^2 m_D^2 \nu^3}{96E} + O(\nu^4). \quad (12)$$

In the last expression the first term corresponds to the magnetic interaction and the second term corresponds to the electric one. Hence, from the above result it is evident that in case of the zero temperature plasma the longitudinal and transverse modes contribute at different order whereas in case of the high temperature plasma there is no such splitting.

Another transport coefficient which could be of importance to study the equilibration property of the quasiparticle is the momentum diffusion coefficient ( $B_{ij}$ ).  $B_{ij}$  is defined via the relation,

$$B_{ij} = \int d\Gamma q_i q_j. \quad (13)$$

$B_{ij}$  can be decomposed into longitudinal ( $B_l$ ) and transverse components ( $B_t$ ) as follows,

$$B_{ij} = B_t \left( \delta_{ij} - \frac{p_i p_j}{p^2} \right) + B_l \frac{p_i p_j}{p^2}. \quad (14)$$

Eventually  $B_{l,t}$  can be obtained by weighting the interaction rate with the longitudinal/transverse momentum transfer in a collision with the plasma. One can proceed in the same way as that of the drag coefficient to obtain  $\mathcal{B}$  in two extreme scenarios ( $T \neq 0, \mu = 0$  and  $T = 0, \mu \neq 0$ ) respectively [3, 6],

$$\begin{aligned}\mathcal{B} &\simeq \frac{e^4 T^3}{18\pi} \left( -\frac{1}{2} + \frac{3}{2} \log \left| \frac{q_{\max}}{m_D} \right| + \frac{1}{2} \log \left| \frac{2}{\sqrt{\pi}} \right| \right) \\ \mathcal{B} &\simeq \frac{e^2 m_D^3 \nu^3}{72\pi} + \frac{e^2 m_D^3 \nu^4}{128} + O(\nu^5).\end{aligned}\tag{15}$$

From the Eqs.(11), (12) and (15) it is seen that physics changes significantly from high to zero temperature in both  $\eta$  and  $\mathcal{B}$ . The logarithmic dependence in high temperature changes to algebraic form in the ultradegenerate limit.

## 4 Summary

The main concern of the present work is to reveal the differences of well known high temperature results with their zero temperature counterparts. Interestingly it is seen that although the calculation of  $\gamma, \eta$  or  $\mathcal{B}$  involve very similar starting point the final result however differ qualitatively mainly because of the presence of the Pauli blocking. For example we find that at zero temperature for the excitations near the Fermi surface leading contributions emanate from the transverse sector while the Coulomb interaction contribute at the subleading order. The infrared behaviour of the quantities we calculate here also exhibit dissimilar infrared behaviour. In fact the usual logarithmic divergence what one encounters become worse with algebraic dependence. In the end however they give finite results as we have shown above.

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